

# EQUITY TERM STRUCTURES WITHOUT DIVIDEND STRIPS DATA

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# Motivation

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# The term structure of equity risk premia

- The term structure of discount rates for risky assets plays an important role in many fundamental economic contexts:
  - Pricing an asset or evaluating an investment opportunity with a specific maturity
  - Investment in climate-change mitigation (extra long maturity)
- Active area of empirical research. Two seminal approaches:
  1. **Lettau and Wachter (2007): use the C-S of equities.**  
Early literature also relied on strong parametric assumptions: Bansal et al. (2005), Hansen et al. (2008), Da (2009).
  2. **Van Binsbergen et al. (2013): use new dividend strips data.**  
Also van Binsbergen et al. (2012), van Binsbergen and Koijen (2015).

# Our approach

- We use the **cross-section of equities and rich dynamics**:
  - Specify an empirical affine model; price all assets jointly
  - Impose discipline: pricing restrictions, state vector choice
  - Generate T-S of ER for market and 100 C-S portfolios
  - Similar to Lettau and Wachter (2007), but much richer well-disciplined SDF dynamics with emphasis on C-S of equities
- Distinct feature – **rich, realistic dynamics**, motivated by recent empirical AP findings:
  1. Kozak, Nagel, and Santosh (JF 2018, JFE 2019): a few dominant PCs of anomaly returns explain the C-S
  2. Haddad, Kozak, and Santosh (RFS 2020): D/Ps of PCs to predict risk prices → critical for adequately capturing SDF dynamics

## Goals and findings

- Model-implied EY match yields in the literature well
  - Strip data since 2004: BMSY, BK
  - We do not use any strip data in our estimation!
- Use model to extend the term-structure data:
  - over time, back to the 1970s;
  - across maturities, e.g., 1–100 years,
  - across portfolios, e.g., small or value stocks.

# Goals and findings

- Term-structure results:
  - Results of the post-2004 sample carry over to the longer sample:
    - T-S inverts in almost all of the additional recessions
    - T-S of forward discount rates is still “too flat” on average
    - Expected div. growth varies substantially over time
  - New cross-sectional results:
    - Value (growth) stocks strongly increasing (flat) T-S of ER
    - Small stocks: flat T-S of ER, large stocks: mildly increasing
    - Time-series: small stocks’ inversion in the late 90s
- Our goal:
  - new “stylized facts” to guide and evaluate AP models

# Methodology

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## Model: Setup

- State vector  $F_t$ :

$$\underbrace{F_{t+1}}_{k \times 1} = \underbrace{c}_{k \times 1} + \underbrace{\rho}_{k \times k} F_t + u_{t+1}, \quad (1)$$

- SDF:

$$m_{t+1} = -r_{f,t} - \frac{1}{2} \lambda_t' \Sigma \lambda_t - \lambda_t' u_{t+1}, \quad (2)$$

where  $\lambda_t = \lambda + \Lambda F_t$ .

- Log prices (returns):

$$\Delta p_{t+1} - r_{f,t} = \gamma_0 + \gamma_1 F_t + \gamma_2 u_{t+1}, \quad (3)$$

- Implies equity prices satisfy:

$$y_t \equiv \log \left( 1 + \frac{D_{t+1}}{P_{t+1}} \right) = b_0 + b_1 F_t. \quad (4)$$



## Model: State vector and restrictions

- $F_t$  contain the market, PCs of L-S anomalies, and their D/Ps:

$$F = [r_M, r_{PC_1}, \dots, r_{PC_3}, y_M, y_{PC_1}, \dots, y_{PC_3}]'$$

- Restrictions:
  1. Only shocks to returns are priced
  2. Only  $y_t$  predict  $F_{t+1}$ , that is,

$$\Lambda = \begin{bmatrix} 0_{4 \times 4} & \tilde{\Lambda}_{4 \times 4} \\ 0_{4 \times 4} & 0_{4 \times 4} \end{bmatrix} \Rightarrow \rho = \begin{bmatrix} 0_{4 \times 4} & \rho_{r,y} \\ 0_{4 \times 4} & \rho_{y,y} \end{bmatrix}.$$

## Model: Other portfolios

- Prices and returns of other well-diversified portfolios are measured with error:

$$y_t = b_0 + b_1 F_t + \epsilon_t$$

$$r_{t+1} - r_{f,t} = \beta_0 + \beta_1 F_t + \beta_2 u_{t+1} + \varepsilon_{t+1}.$$

- Only  $b_0, b_1, \beta_2$  need to be estimated (easy);  $\beta_0, \beta_1$  pinned down by no-arbitrage

# Model: Recap

- Affine model
- State vector contains 4 PCs of anomaly ret. and their D/Ps
- Dynamics and restrictions motivated by:
  1. Kozak, Nagel, and Santosh (JF 2018, JFE 2019):
    - No near-arbitrage  $\Rightarrow$  only a few large PCs show up as factors
    - Reduces C-S of many factors to a few dominant PC-factors
  2. Haddad, Kozak, and Santosh (RFS 2020):
    - Bound cond. SR  $\Rightarrow$  only dominant PCs should be *predictable*
    - Forecast  $E_{t-1}[R_t]$  on PC-factors using own D/Ps
    - Important source of SDF time-variation
- These choices are critical for realistic SDF dynamics

## Model: Estimation

- Joint GMM estimation with these moments:
  - State space shocks:  $u_{t+1} \perp F_{y,t}$
  - Portfolio returns:  $r_{t+1} \perp \{F_{y,t}, u_{t+1}\}$
  - Yields:  $y_t \perp F_{y,t}$
- Asymptotic GMM standard errors
  - Spectral covariance matrix with 12 lags
  - Standard errors of everything via the Delta method

## Dividends strips prices and equity yields

- Dividend strips prices:

$$\frac{P_t^{(n)}}{P_t} = \mathbb{E}^{\mathbb{Q}} \left[ \frac{D_{t+n}}{P_t} \right] = e^{a_{n,1} + d_{n,1}F_t} - e^{a_{n,2} + d_{n,2}F_t},$$

where  $a_{n,\cdot}, d_{n,\cdot}$  are given by recursions.

- Equity yields (no approximations):

$$e_{t,n} = \frac{1}{n} \log \left( \frac{D_t}{P_t^{(n)}} \right) = \frac{1}{n} \left[ \log (e^{y_{t+n}} - 1) - \log \left( \frac{P_t^{(n)}}{P_t} \right) \right].$$

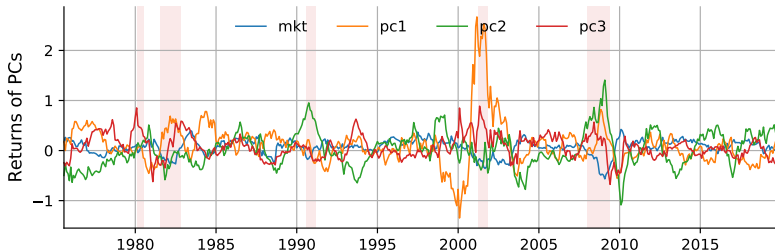
- Similarly, can compute:
  - Realized and expected HPR returns on dividend strips
  - Decomposition into hold-to-maturity (HTM) exp. returns and expected real div. growth rates at each maturity
  - Volatility, Sharpe ratios, forward equity yields, etc.

# Results

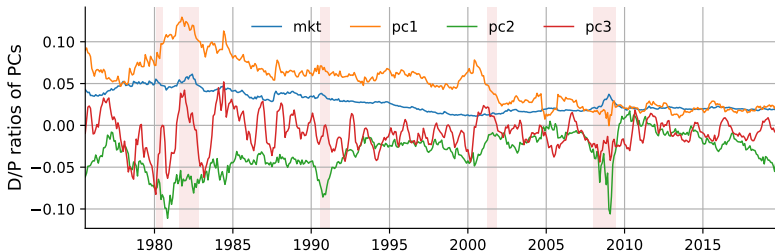
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- Stocks sorted into 3 portfolios based on 50 characteristics from Kozak (2019), one at a time
- Joint GMM estimation, asymptotic GMM s.e.
- All results orthogonal to the bond term-structure
- Monthly sample from September 1974 to December 2019. Annual horizon.

# PC-factor returns predictability and predictors' dynamics



(a) PC factor returns



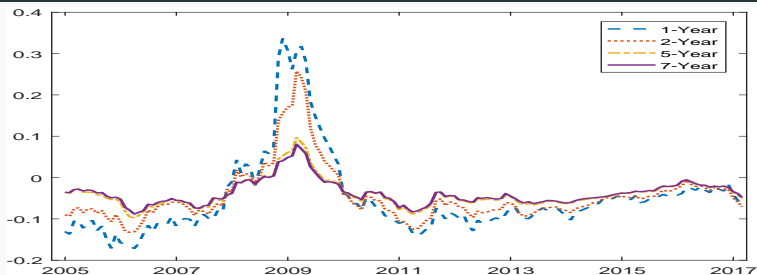
(b) D/P ratios of SDF factors



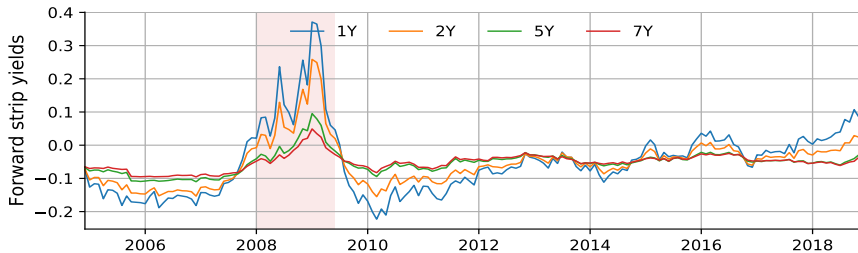
## **Fit to traded S&P 500 futures**

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# Time-series of equity yields: our model vs. strips data (2005–)

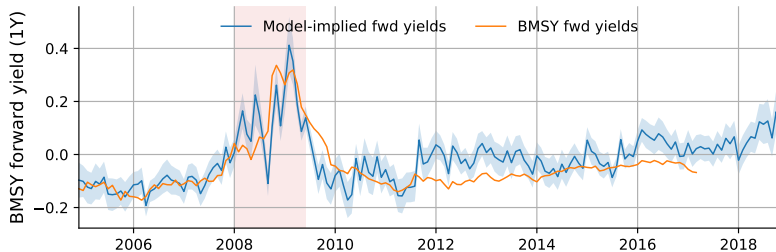


(a) Bansal, Miller, Song, Yaron (2018): equity fwd yields for S&P 500

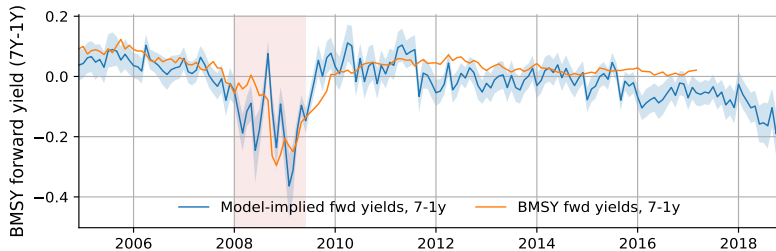


(b) Model-implied fwd equity yields for large stocks (approx. S&P 500)

# Model-implied fwd equity yields vs. fwd equity yield data

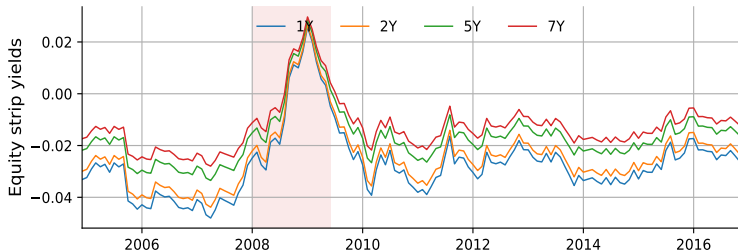


(a) 1-year

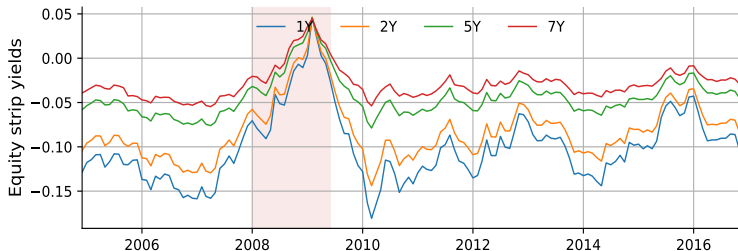


(b) 7y-1y

# Dynamics of benchmark-implied equity strip yields

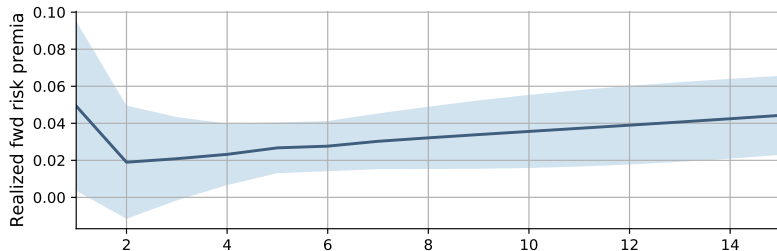


(a) CAPM

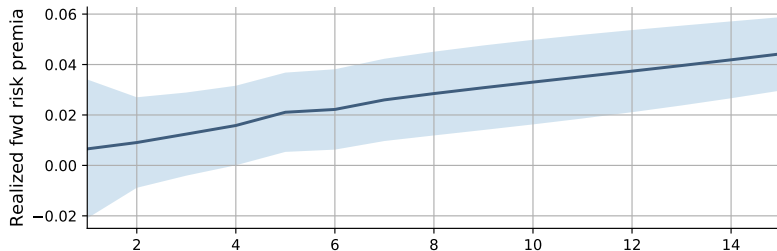


(b) FF 5-factor + Momentum

# Estimated term structure of forward strip risk premia

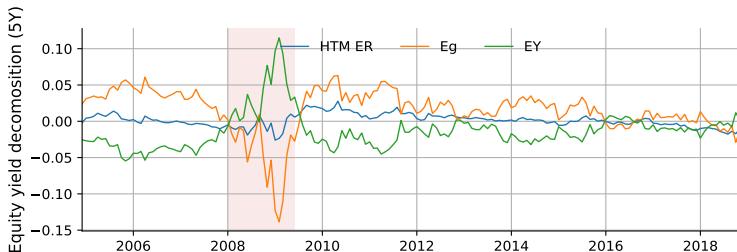


(a) BMSY sample (2005–2019)

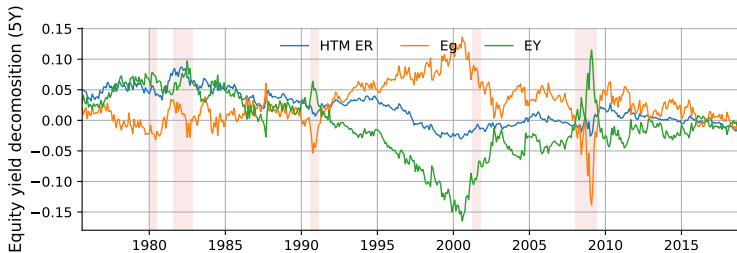


(b) Full sample (1975–2019)

# Decomposition of the 5-year equity yield

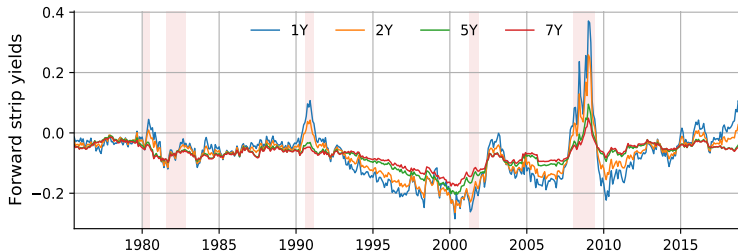


(a) BMSY sample (2005–2019)

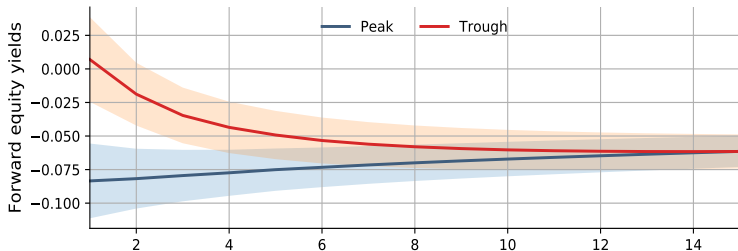


(b) Full sample (1975–2019)

# Model-implied fwd equity yields (FEY)



(a) FEY dynamics for selected maturities



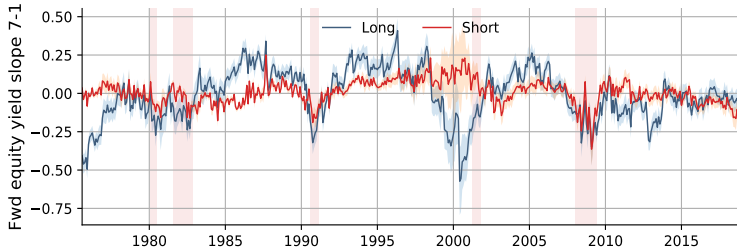
(b) FEY conditional on NBER recessions

## Cross-sectional results

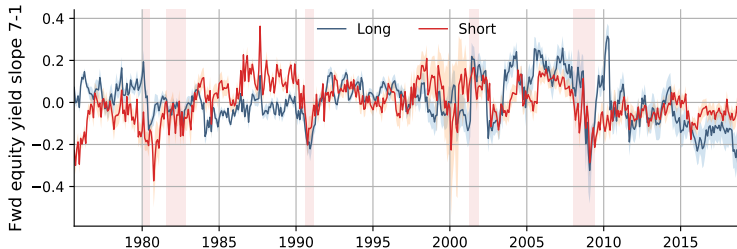
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# Slope 7-1 of forward equity yields

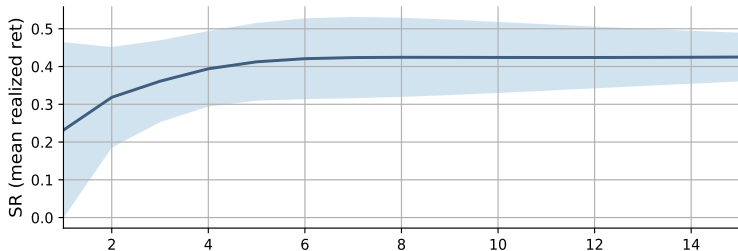


(a) Small and large stocks

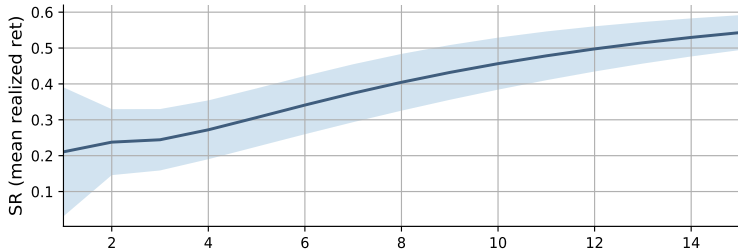


(b) Value and growth stocks

# Sharpe ratios: value vs. growth

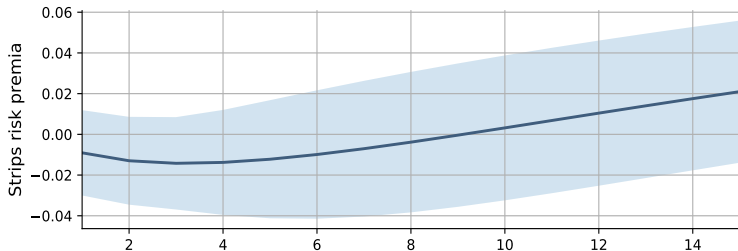


(a) Growth stocks

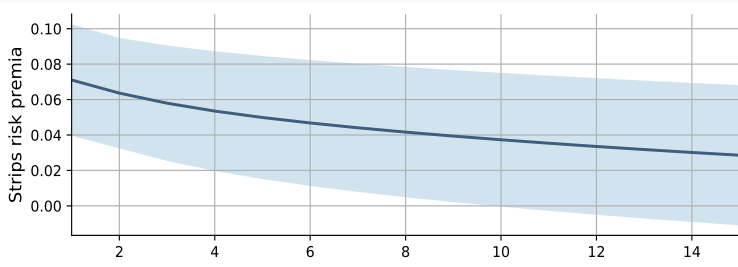


(b) Value stocks

# Expected returns on long-short portfolios

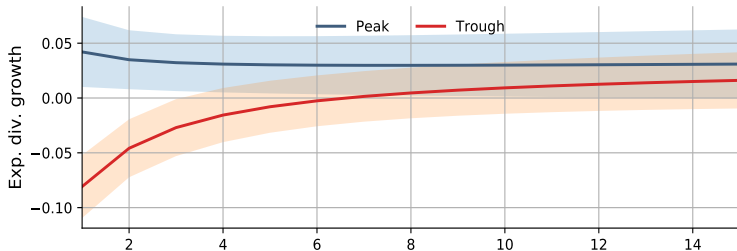


(a) Value-growth

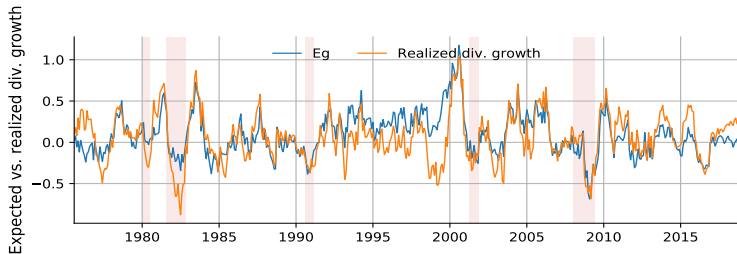


(b) Small-big

# Implied expected dividend growth

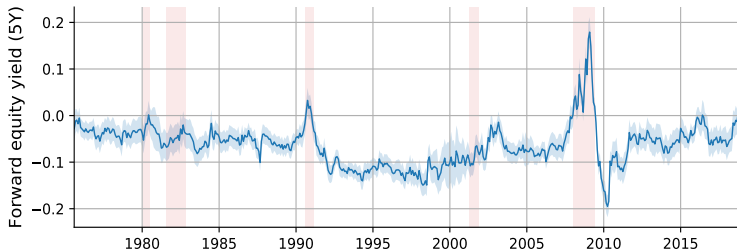


(a) Large stocks (S&P 500)

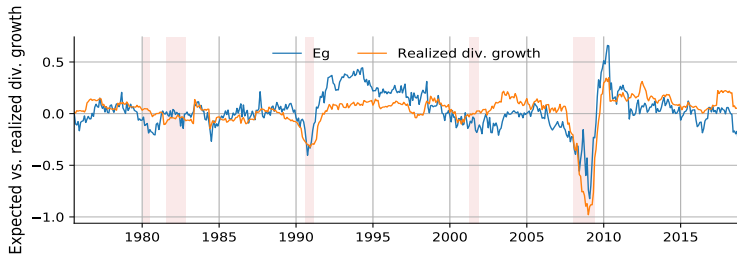


(b) Momentum stocks

# High leverage stocks: yields and expected div. growth



(a) FEY



(b) Expected dividend growth

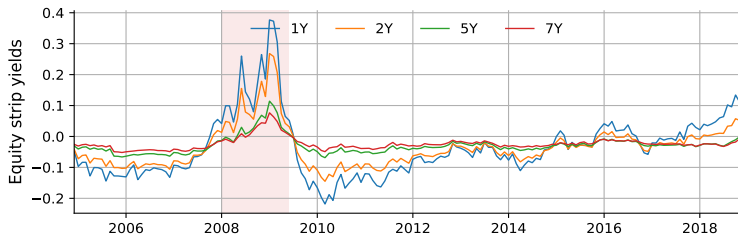
# Conclusions

- New methodology to study equity price dynamics and the term structure of risk premia
- We produce new “stylized facts”:
  - The T-S and C-S behavior of dividend term structures
  - Similar to VAR by Sims (1980) we provide new moments for evaluation and guidance of AP models
  - Our synthetic strips extend the term-structure data:
    - over time, back to the 1970s;
    - across maturities, e.g., 1–100 years,
    - across portfolios, e.g., small or value stocks.
- Potential applications:
  - Test C-S implications of models: Hansen et al. (2008), Belo (2010); Kogan and Papanikolaou (2013, 2014)
  - Evaluate investments of different horizon, e.g., PE.

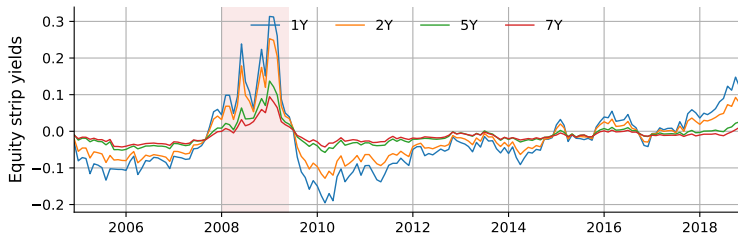
## **Appendix: Out-of-sample analysis**

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# IS and OOS dynamics of model-implied yields in BMSY sample



(a) In-sample



(b) Out-of-sample

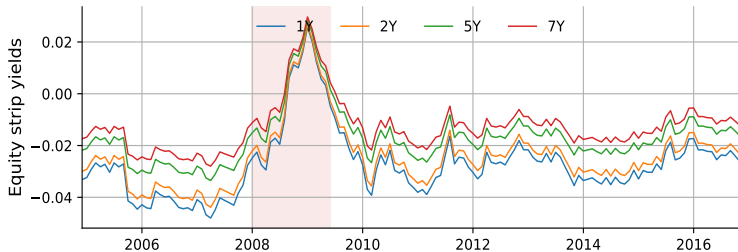
**Figure 4:** Model parameters are estimated in the 1975–2004 sample and held constant throughout the rest of the sample.



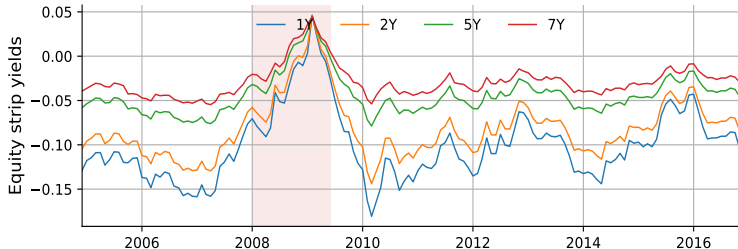
## **Appendix: Counterfactual analysis**

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# Dynamics of benchmark-implied equity strip yields

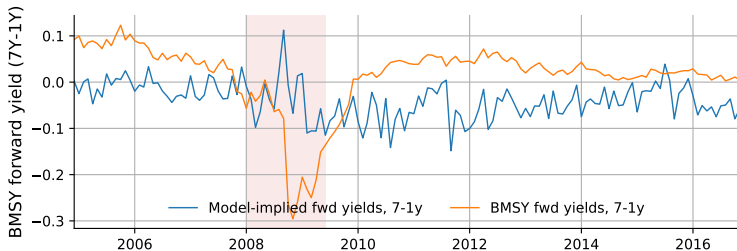


(a) CAPM

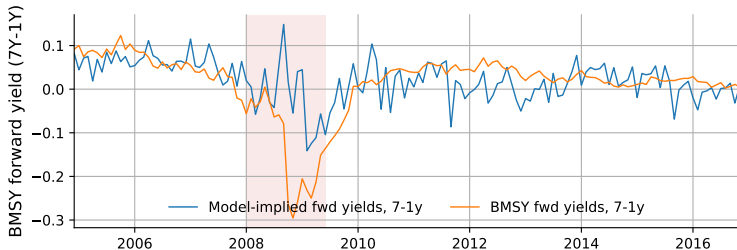


(b) FF 5-factor + Momentum

# Benchmark-implied FEY vs. data



(a) CAPM



(b) FF 5-factor + Momentum

## **Appendix: Motivation**

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# Characterizing the SDF: From stocks to managed portfolios

- Project the “true” SDF onto the space of excess returns,

$$M_t = 1 - b'_{i,t-1} (R_{i,t} - E[R_{i,t}]),$$

- Let  $b_{i,t-1}$  be linear in C-S characteristics  $X_{i,t-1}$ :

$$b_{i,t-1} = X_{i,t-1}\gamma,$$

- compresses  $N$  coeffs for each stock to  $K$  coeffs for each char.
- $\gamma \equiv \gamma_{t-1}$  contains only remaining time-varying *aggregate* info
- Plug in:

$$M_t = 1 - \gamma'_{t-1} (F_t - E[F_t]),$$

- where  $F_t = X'_{t-1}R_t$  is a vector of characteristics-based factors
- i.e., we can equivalently represent SDF in terms of managed portfolios

# Characterizing the SDF: Two results

1. Kozak, Nagel, and Santosh (JF 2018, JFE 2019):
    - Absence of near-arbitrage requires only a few large PCs show up as factors
    - Reduces the C-S of potentially hundreds of factors to a few dominant PC-factors
  2. Haddad, Kozak, and Santosh (RFS 2020):
    - Bound cond. SR: only dominant PCs should be predictable
    - Forecast  $E_{t-1}[R_t]$  on PC-factors using own valuation ratios
    - PC exp. returns are highly predictable, more than agg. market
    - Important source of SDF time-variation!
    - recover  $\gamma_{t-1}$  and prices of risk  $\Rightarrow$  SDF
- $\rightarrow$  We estimate an empirical model motivated and consistent with these findings

## References

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